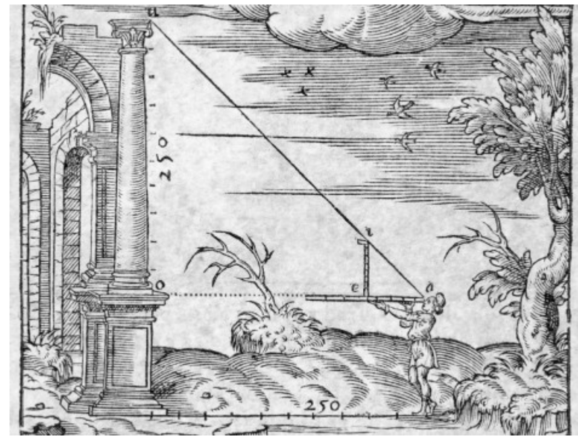
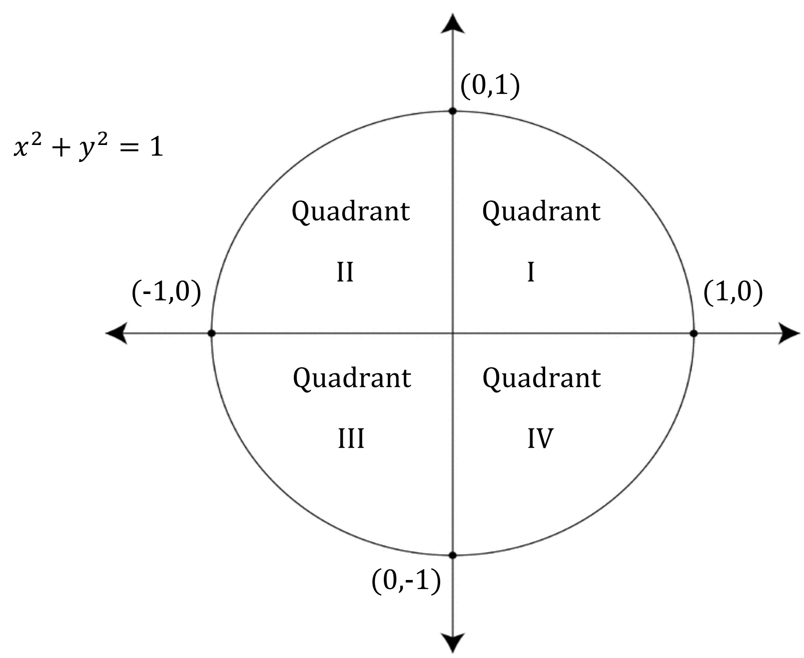
6.1: Trig Basics

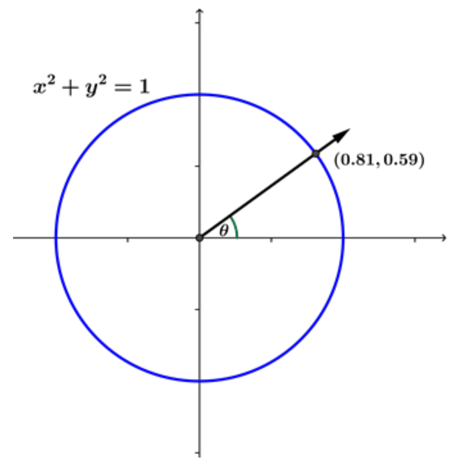
The Unit Circle



To begin our investigation of trigonometry and music, we first revisit the unit circle, which you should remember from geometry. The unit circle is **a circle with radius 1 that is centered at the origin.** It is split up into four quadrants (as shown below). Remember that the equation of the unit circle is $x^2 + y^2 = 1$. This means that every point on the unit circle must fit into this equation.



1. A frog is sitting on the edge of a playground carousel with radius 1 meter. The ray through the frog's position and the center of the carousel makes an angle of measure θ with the horizontal, and his starting coordinates are approximately (0.81,0.59). Find his new coordinates after the carousel rotates by each of the following amounts.



   a. $\frac{\pi}{2}$

   b. π

   c. 2π

   d. $-\frac{\pi}{2}$

   e. $-\pi$

   f. $\frac{\pi}{2} - \theta$

   g. $\pi - 2\theta$

   h. -2θ

   i. If the point $P(x, \frac{\sqrt{3}}{2})$ is on the unit circle, find the x-coordinate of the point in Quadrant I.

Keep that point in mind! That is one of the common points you will often use when working with the unit circle.

II. Terminal Points

Given a real number t, suppose we want to find the point on the unit circle that is a distance t from the point $(1, 0)$ along the circumference of the circle. If t is positive, we move in the counterclockwise direction. If t is negative, we move in the clockwise direction. The point $P(x, y)$ that is found is called the terminal point determined by t.

Discovering the important terminal points:

For the next few questions, keep in mind that the circumference of the unit circle is 2π.

2. If $t = \frac{\pi}{2}$, we have traveled $\frac{1}{4}$ of the circumference ($\frac{1}{4}$ of 2π is $\frac{\pi}{2}$). So the terminal point is: ______________.

3. If $t = \pi$, we have traveled half of the circumference. So the terminal point is: _______.

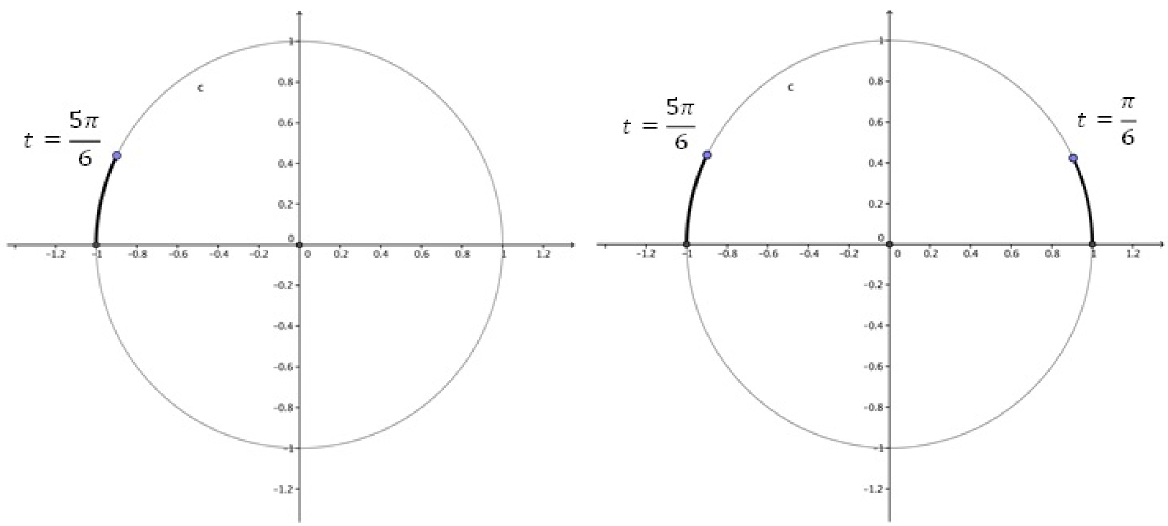
4. If $t = \frac{3\pi}{2}$, we have traveled $\frac{3}{4}$ of the circumference. So the terminal point is: ______________.

For other values of t, it is not as easy to find the terminal points. However, there are a few common terminal points that should be memorized, as you will frequently use them. These terminal points are listed below, along with their corresponding t values.

Value of t	Terminal Point
0	$(1,0)$
$\frac{\pi}{6}$	$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
$\frac{\pi}{4}$	$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$\frac{\pi}{3}$	$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
$\frac{\pi}{2}$	$(0,1)$
π	$(-1,0)$
$\frac{3\pi}{2}$	$(0,-1)$

III. Reference Numbers

The **reference number** $\bar{t}$ associated with t is the shortest distance from the terminal point determined by t to the x-axis. For example, let's look at the terminal point determined by $t = \frac{5\pi}{6}$ (shown below).



Notice how the terminal point determined by $t = \frac{5\pi}{6}$ is the same distance away from the x-axis as the terminal point determined by $t = \frac{\pi}{6}$.

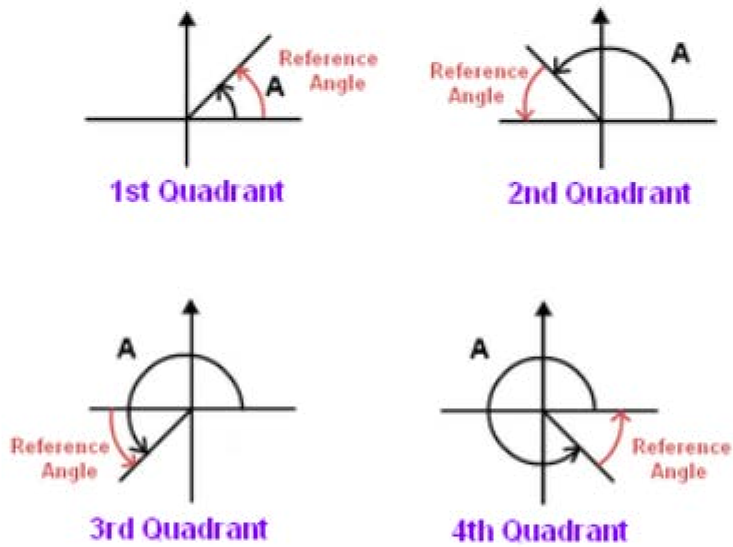
Therefore, the reference number for the value $t = \frac{5\pi}{6}$ is $\bar{t} = \frac{\pi}{6}$.

<u>Your Turn</u>: Find the reference numbers for the following values of t.

5. $t = \frac{3\pi}{4}$

6. $t = \frac{5\pi}{3}$

7. $t = \frac{7\pi}{6}$



Of course, you will use the reference number to find the terminal points in other quadrants, as well. Once you know the reference number, use the terminal point associated with it and determine whether the signs are positive or negative. For example, we know that the reference number associated with $t = \frac{5\pi}{6}$ is $\bar{t} = \frac{\pi}{6}$. The terminal point of $t = \frac{\pi}{6}$ (one of our common t values) is $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. However, since $t = \frac{5\pi}{6}$ is in Quadrant II, where the x-coordinate is negative and the y-coordinate is positive, its terminal point is $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$.

<u>Your Turn</u>: Find the terminal point of the following values of t by first finding the reference number.

8. $t = \frac{7\pi}{4}$

9. $t = -\frac{5\pi}{6}$

10. $t = \frac{19\pi}{6}$

11. The figure below is a unit circle filled in with some of the common t values and their corresponding terminal points. The *t* values are given in both radian and degree measurements. Notice how terminal points in Quadrants II are the same as those in Quadrant I, except for the different signs. Complete the unit circle for the other values